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Design of Compensator of Approximation of Large Time Delay Systems via Reduced Order Model

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Abstract

The present paper deals with a approximating method for large time – delays of multi-input multi-output (MIMO) dynamical systems. Time delay terms of the state space equations are described by delay matrix in the complex domain. A mixed model reduction method of matrix Pade-type-Routh model for the multivariable linear systems was presented. Matrix Pade-type Routh model approximation can largely reduce the instability and the overshoot, so the fast response property is improved. Simulation results of the proposed method are presented to illustrate the correctness and effectively.

Keywords: Multi-input multi-output Systems; Time-Delays; matrix Pade-type model reduction, Routh table.

Introduction

A time delay in input-output relations is a common property of many industrial processes control [1], [2], such as thermo technical processes, chemical processes etc. The effects of time delay are essential. Take a freeze dryer for example, the temperature control system is a first order large inertia system produce dynamic temperature fluctuations, which lead the freeze dried products cannot fulfil the high quality demand. The time-delay property should not be neglected, that when unknown greatly complicates the control problem. In the analysis of a high degree multivariable system, it is often necessary to compute a lower degree model so that it may be used for a analogue or digital simulation of the system. The denominator polynomial of the reduced model is obtained from the Routh table and its numerator matrix polynomial is obtained by the matrix Pade-type Routh Model [6],[7].However, majorities of these ways engage in the analysis of single time-delay variable. Pade-type Routh model is popular method to approximate a scalar pure delay exponential function In this paper, the multi-input multi-output multivariable matrix Pade-type approximation, the basic concept is defined and applied to the state-space approximation problem of multivariable linear systems.

This paper has five sections, section II states matrix Pade-type-Routh model reduction method. Section III explains the state equation of MIMO delay system. Section VI presents root locus lead compensator design. Section V presents two simulation examples and one comparison example with different large time delay based on the proposed method, the step responses are plotted. Section VI gives the conclusion.

Matrix Pade-Type-Routh Model Reduction Method

Let the transfer function of a higher order system be represented by [6], [7]

$$G(s)=\frac{D_0+D_1 s+\ldots+D_{n-1}s^{n-1}}{e_0+e_1 s+\ldots+e_n s^n}=\frac{D_n(s)}{E_n(s)}, \quad (1)$$

Where D_i, i=0,1,…, n-1 are constant l x r matrices, and e_i, i=0,1,…, n are scalar constants. G (s) can be expanded into a power series of the from

$$G(s)=C_0+C_1 s+C_2 s^2+\ldots. \quad (2)$$

Where the C_i, i=0, 1… are l x r constant matrices which satisfy the relation

$$C_0=\frac{1}{e_0}D_0,$$

$$C_i=\frac{1}{e_0}[D_i-\sum_{j=0}^{i-1}e_{i-j}C_j],\ i=1,2..... \quad (3)$$

Thus using Eq. (3) the matrix transfer function may be expanded into a power series.

Assume that the reduced model R (s) of order n is required, and let it be of the form

$$R(s)=\frac{D_k(s)}{E_k(s)}=\frac{D_0+D_1s+...+D_{k-1}s^{k-1}}{e_0+e_1s+...+e_{k-1}s^{k-1}+e_ks^k}, \quad (4)$$

where the D_i, i=0,1,…, k -1 are constant l x r matrices, and e_i, i=0,1,…., k are scalar constants.

Algorithm 1

Step 1 The denominator E_k (s) of reduced model transfer function can be constructed from the Routh Stability array of the denominator of the system transfer function as follows.

The Routh stability array is formed by the following

$$b_{i,j}=b_{i-2,j+1}-\frac{b_{i-2,1}b_{i-1,j+1}}{b_{i-1,1}}, \quad (5)$$

$$where\, i\geq 3\, and\, 1\leq j\leq\left[\frac{(k-i+3)}{2}\right]$$

The Routh table for the denominator of the system transfer function is given as

$$\begin{array}{llll} b_{11}=e_n & b_{12}=e_{n-2} & b_{13}=e_{n-4} & b_{14}=e_{n-6} \ ... \\ b_{21}=e_{n-1} & b_{22}=e_{n-3} & b_{23}=e_{n-5} & b_{24}=e_{n-7} \ ... \\ b_{31} & b_{32} & b_{33} & ..... \\ ..... & & & \\ b_{k-1,1} & b_{k-1,2} & & \\ b_{k,1} & & & \\ b_{k+1,1} & & & \end{array} \quad (6)$$

E_k (s) may be easily constructed from the (n+1-k)-th and (n+2-k)-th and (n+2-k)-th rows of the above to give

$$E_k(s)=\sum_{j=0}^{n}b_js^j$$
$$=b_{k+1-n.1}s^n+b_{k+2-n.1}s^{n-1}+b_{k+1-n.2}s^{n-2}+.... \quad (7)$$

Step 2 the numerator D_n (s) of reduced model transfer function by (5) and (6) can be obtained from

$$D_k(s)=s^{n-1}\phi\left(\frac{\tilde{E}_k(x)-\tilde{E}_k(s^{-1})}{x-s^{-1}}\right), \quad (8)$$
$$where\, \tilde{E}_k(s)=s^nE_k(s^{-1}).$$

Thus the reduced model transfer function is given by

$$R(s)=\frac{D_k(s)}{E_k(s)}$$

State Equation of Mimo Delay System

Consider a MIMO continuous-time system with delays

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \vdots\\ \dot{x}_n\end{bmatrix}=\begin{bmatrix}a_{11} & a_{12} & \cdots & a_{1n}\\ a_{21} & a_{22} & \cdots & a_{2n}\\ \cdots & \cdots & \cdots & \cdots\\ a_{n1} & a_{n2} & \cdots & a_{nn}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ \vdots\\ x_n\end{bmatrix}+\begin{bmatrix}b_{11} & b_{12} & \cdots & b_{1m}\\ b_{21} & b_{22} & \cdots & b_{2m}\\ \cdots & \cdots & \cdots & \cdots\\ b_{n1} & b_{n2} & \cdots & b_{nm}\end{bmatrix}\begin{bmatrix}u_1(t-\tau_1)\\ u_2(t-\tau_2)\\ \vdots\\ u_m(t-\tau_m)\end{bmatrix} \quad (9)$$

$$\begin{bmatrix}y_1\\ y_2\\ \vdots\\ y_m\end{bmatrix}=\begin{bmatrix}c_{11} & c_{12} & \cdots & c_{1n}\\ c_{21} & c_{22} & \cdots & c_{2n}\\ \cdots & \cdots & \cdots & \cdots\\ c_{m1} & c_{m2} & \cdots & c_{mn}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ \vdots\\ x_m\end{bmatrix} \quad (10)$$

$$\dot{x}=Ax+Bu \quad (11)$$

$$y=Cx \quad (12)$$

Where

$x\in R^n, u\in R^m, y\in R^1, and\, A\in R^{nxn}, B\in R^{mxm}, C\in R^{mxn}\, are$ the situation of input and output vectors, respectively. Laplace transform of Eq. (11) and Eq. (12) respectively, then the transform function matrix of the MIMO system with delays can be obtained

$$\begin{aligned}Y(s)&=C(sI-A)^{-1}B\tau(s)U(s)\\ &=G_1(s)G_2(s)U(s)\\ &=G(s)U(s)\end{aligned} \quad (13)$$

Where $G_1(s)$, $G_2(s)$, are without and with time delay parts of MIMO system G(s), τ (s) is pure delays diagonal matrix which is given by

$$G_2(s)=\tau(s)=\begin{bmatrix}e^{-\tau_1 s} & 0 & \dots & 0\\ 0 & e^{-\tau_2 s} & \dots & 0\\ \dots & \dots & \dots & \dots\\ 0 & 0 & 0 & e^{-\tau_m s}\end{bmatrix} \quad (14)$$

Thus the time delay is represented in transfer function form as:

$$e^{-\tau s}=\frac{2-\tau s}{2+\tau s} \quad (15)$$

Design of Root Locus Lead Compensator

Step 1: Determine damping factor δ to give the desired overshoot and resonant frequency ω_n to give the desired speed of response to the closed loop system.

Step 2: At the desired pole position (P_d) determined by the δ and ω_n of step l determine $\angle G_f(P_d)$

Then $\angle G_c(P_d)=\pm 180^0-\angle G_f(P_d)$

Step 3 : Add the compensator pole - zero pair so that $\angle G_c(P_d)$ is as determined in step 2 and place the compensator zero such that the resulting root locus will have all poles beside P_d and P_d either far into the

LHP or near zeroes. Usually this means canceling out the plant pole nearest (but not on) the jw axis on the negative real axis.

Step 4: With the spi rule determine K, which for the compensated closed loop system will give a pole at P_d. Determine compensator gain A to give this K.

Step 5: Check the time response to see that the desired overshoot and speed of response have been obtained. If not go back to step 3 and adjust the position of the compensator zero so that the desired overshoot and speed of response have been obtained. If this adjustment does not result in the desired overshoot and speed of response, return to step 1, and adjust δ and ω_n in the direction required to give a more desirable response.

Simulation Example

Consider MIMO continuous-time system with delays

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \begin{bmatrix}2 & 1 & -8\\ 3 & 4 & -4\\ 0 & 1 & -7\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \begin{bmatrix}4 & 5 & 6\\ 1 & 0 & 1\\ 3 & 1 & 1\end{bmatrix}\begin{bmatrix}u(t-32)\\ u(t-100)\\ u(t-800)\end{bmatrix}$$

$$\begin{bmatrix}y_1\\ y_2\\ y_3\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$$

Input time delays are

$\tau_1 = 32\,s$, $\tau_2 = 100\,s$, $\tau_3 = 800\,s$, respectively.

$Y(s) = C\,(sI-A)^{-1}\,B\,\tau(s)\,U(s)$

$$\frac{Y(s)}{U(s)} = G_1(s)G_2(s)$$

Where G_1 (s) is linear MIMO System

G_2 (s) is Purely time delay

$$G_2(s) = \tau(s)$$

Dut to pure delays component $G_2(s)$ is a diagonal matrix similarity transformation approach is used to obtain the decoupled state space equation, such that each output is corresponding to one input. Fig.1, Fig.2 and Fig.3 gives step response.

$$\tau(s) = \begin{bmatrix}e^{-32s} & 0 & 0\\ 0 & e^{-100s} & 0\\ 0 & 0 & e^{-800s}\end{bmatrix}$$

$$G_1(s) = \frac{\begin{bmatrix}s^2-9s+6 & 2s^2+6s+28 & -8(1+s)\\ s^2-20s-9 & 6-12s & s^2-7s-8\\ 2s^2+s+1 & s^2+2 & s^2+s\end{bmatrix}}{s^3+7s^2+14s+8}$$

By applying pade – type Routh model the order reduced system transfer function is obtained as follows:

Reduced order denominator (by applying Routh table):

s^3	1	14
s^2	7	8
s^1	12.85	0
s^0	8	

$E_2(s) = 7s^2+12.85s+8$

Reduced order numerator (by applying pade –type method):

$$C_0 = \frac{1}{e_0}D_0 = \begin{bmatrix}0.75 & 3.5 & -1\\ -1.125 & 0.75 & -1\\ 0.125 & 0.25 & 0\end{bmatrix}$$

$$C_1 = \frac{1}{e_0}[D_1 - e_1C_0] = \begin{bmatrix}-2.437 & -5.375 & 0.75\\ 0.531 & -2.8125 & 0.875\\ 0.093 & -0.437 & 0.125\end{bmatrix}$$

$$D_0 = e_0\,c_0 = \begin{bmatrix}6 & 28 & -8\\ -9 & 6 & -8\\ 1 & 2 & 0\end{bmatrix}$$

$$D_1 = e_0c_1 + e_1\,c_0 = \begin{bmatrix}-9.851 & 1.975 & -6.85\\ -10.208 & -12.862 & -5.85\\ 2.35 & -0.283 & 1\end{bmatrix}$$

Thus the reduced order transfer function is

$$R_2(s) = \frac{D(s)}{E_2(s)}$$

$$\therefore R_2(s) = \frac{\begin{bmatrix}6-9.851s & 28+1.98s & -8-6.85s\\ -9-10.2s & 6-12.86s & -8-5.85s\\ 1+2.35s & 2-0.283s & s\end{bmatrix}}{7s^2+12.85s+8}$$

By the addition of time delay to the original linear transfer function is

$$T(s) = \frac{Y(s)}{U(s)} = G_1(s)\,G_2(s)$$

$$T(s) = \frac{\begin{bmatrix}-s^3+8.9s^2-16s+0.03 & -s^3-5.9s^2+28s-0.05 & 8s^2+7.9s-0.01\\ -s^3+20s^2+8.9s-0.01 & 12s^2-6s+0.01 & -s^3+7s^2+7.9s-0.01\\ -2s^3-7s^2-s+0.002 & -s^3+0.002s^2-2s+0.004 & -s^3-s^2+0.002s\end{bmatrix}}{s^4+7s^3+14s^2+8s+0.016}$$

Reduced order denominator (by applying Routh Table): $E_2'(s) = 12.67s^2+8.019s+0.016$

Reduced order numerator (by applying Pade-type method) :

$$D'(s) = \begin{bmatrix}0.03-16s & -0.05+28.04s & -0.01+8s\\ -0.01+7.9s & 0.012+6s & -0.016+8s\\ 0.002-s & 0.004+2.4s & 0.002s\end{bmatrix}$$

Thus the reduced order transfer function with time delay is

$$R_2'(s) = \frac{\begin{bmatrix}0.03-16s & -0.05+28.04s & -0.01+8s\\ -0.01+7.9s & 0.012+6s & -0.016+8s\\ 0.002-s & 0.004+2.4s & 0.002s\end{bmatrix}}{12.67s^2+8.019s+0.016}$$

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The simulation results for the original and reduced order systems can be seen from Fig.1, Fig.2 and Fig.3. These are the step responses with time delay for the original transfer function.

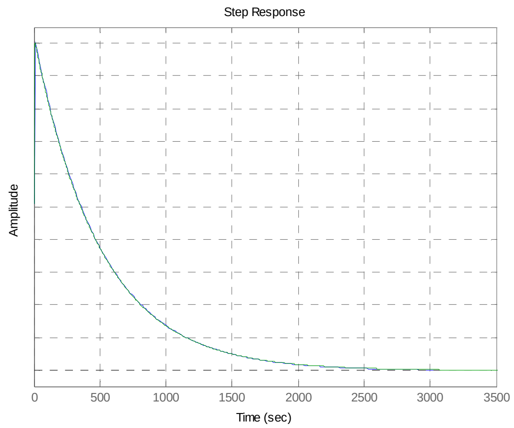

Fig1 . Step response of first output with time delay.

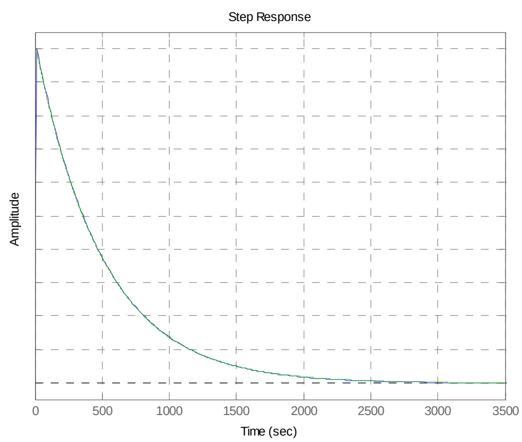

Fig2.Step response of second output with time delay.

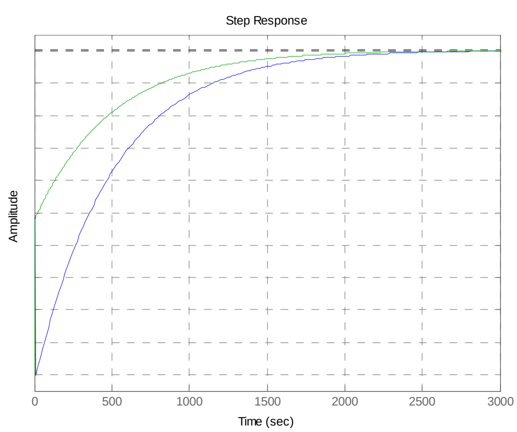

Fig3. Step response of third output with time delay.

And we can observe the step responses for original and reduced order system without time delay in Fig.4, Fig.5 and Fig.6.

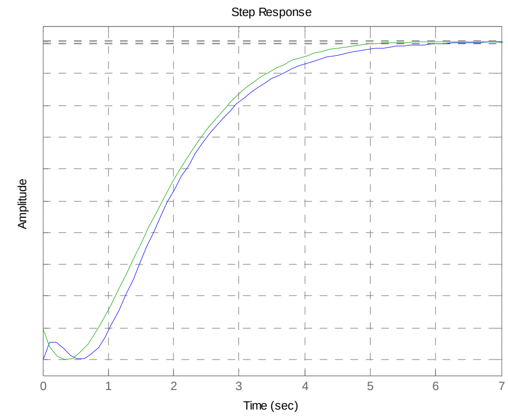

Fig4.Step response of first output without time delay

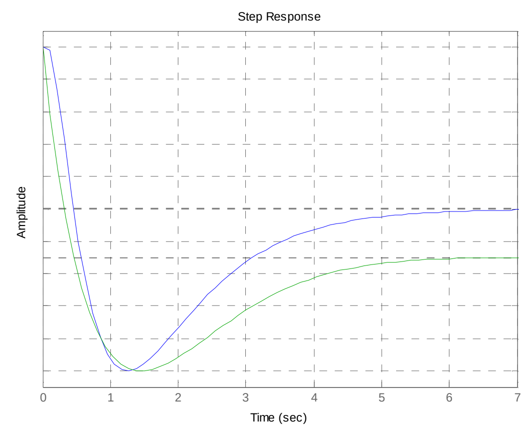

Fig5. Step response of second output without time delay

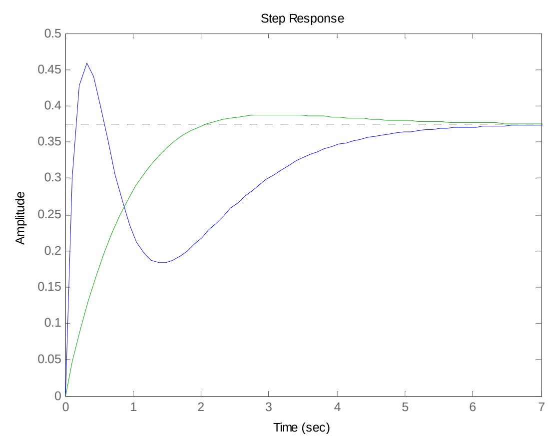

Fig6.Step response of third output without time delay

Example :2 Consider the 8th order system as follows:

$$G(s)=\frac{\begin{bmatrix} g_{11}(s) & g_{12}(s) & g_{13}(s) \\ g_{21}(s) & g_{22}(s) & g_{23}(s) \end{bmatrix}}{D(s)}$$

$$g_{11}(s)=0.0114s^7+0.5119s^6+2.4152s^5+4.918s^4+6.2164s^3+4.6146s^2+1.7134s+0.261$$
$$g_{12}(s)=-0.0193s^7+0.1121s^6+0.1114s^5+0.4198s^4+0.3241s^3+0.3198s^2+0.1618s+0.0134$$
$$g_{13}(s)=-0.0021s^7+0.202s^6+0.8854s^5+1.3158s^4+1.2115s^3+0.837s^2+0.3148s+0.0142$$
$$g_{21}(s)=0.1936s^7+0.6369s^6+2.1686s^5+4.1829s^4+5.1619s^3+3.1716s^2+1.0559s+0.1246$$
$$g_{22}(s)=0.01988s^7-0.0205s^6-0.2744s^5+1.4091s^4+1.3824s^3+1.1239s^2+0.7439s+0.404$$
$$g_{23}(s)=0.174s^7+0.922s^6+2.5345s^5+4.5005s^4+4.509s^3+2.801s^2+1.2165s+0.1525$$
$$D(s)=s^8+9.83s^7+36.616s^6+65.852s^5+73.018s^4+50.03s^3+17.104s^2+1.919s+0.25$$

with input time delays as follows:

$\tau_1=28s$, $\tau_2=64s$, $\tau_3=128s$, $\tau_4=256s$, $\tau_5=512s$, $\tau_6=200s$, $\tau_7=600s$, $\tau_8=800s$

Reduced order denominator (By applying Routh table):

s^8	1	36.616	73.018	17.104	0.25
s^7	9.83	65.852	50.03	1.919	
s^6	29.91	67.928	16.908	0.25	
s^5	43.52	44.47	1.837		
s^4	37.365	15.64	0.25		
s^3	26.25	1.546			
s^2	13.419	0.25			
s^1	1.056				
s^0	0.25				

Thus the reduced order denominator is obtained as follows:

$$E_2(s)=13.419s^2+1.056s+0.25$$

Reduced order numerator (by applying Pade-Type Method):

$$C_0=\frac{1}{e_0}D_0=\begin{bmatrix} 0.8652 & -0.0536 & 0.0568 \\ 0.8584 & 0.1616 & 0.61 \end{bmatrix}$$

$$C_1=\frac{1}{e_0}(D_1-e_1C_0)$$
$$=\begin{bmatrix} 0.2096 & 1.0552 & 1.032 \\ 2.982 & 1.7348 & 0.1816 \end{bmatrix}$$

$$P_0=e_0^{'}C_0=\begin{bmatrix} 0.2163 & -0.0134 & 0.0142 \\ 0.2146 & 0.0404 & 0.1525 \end{bmatrix}$$

$$P_1=e_0^{'}C_1+e_1^{'}C_0$$

$$P_1=\begin{bmatrix} 0.966 & 0.207 & 0.317 \\ 1.651 & 0.604 & 0.689 \end{bmatrix}$$

Thus the reduced order transfer function is

$$R_2(s)=\frac{P(s)}{E_2(s)}$$

$$R_2(s)=\frac{\begin{bmatrix} 0.966s+0.2163 & 0.207s-0.0134 & 0.317s+0.0142 \\ 1.651s+0.2146 & 0.604s+0.0404 & 0.689s+0.152 \end{bmatrix}}{13.419s^2+1.056s+0.25}$$

The reduced order transfer function (for 1st output) is as follows:

$$R(s)=\frac{1.49s+0.2171}{13.419s^2+1.056s+0.25}$$

(For 1st output)

$$R(s)=\frac{2.944s+0.407}{13.419s^2+1.056s+0.25}$$

(For 2nd output)

By adding compensator to the reduced order system (Root locus lead compensator):

$$R(s)=\frac{0.111(s+0.145)}{s^2+0.0786s+0.0186}*\frac{(s+z_c)}{(s+p_c)}$$

(For 1st output)

$$R(s)=\frac{0.219(s+0.138)}{s^2+0.0786s+0.0186}*\frac{(s+z_c)}{(s+p_c)}$$

(For 2nd output)

And the poles of the system are given as follows:

$-0.0393 \pm j0.135$

$$s_d=-\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$$
$$s^2+2\xi\omega_n s+\omega_n^2=s^2+0.0786s+0.0186$$
$$\omega_n=0.136$$
$$\xi=0.289$$

The desired value of the reduced order system

$$\xi=0.8$$

be

The values of the compensated system are given as:

$$z_c=-0.13 \quad and \quad p_c=-0.164$$

Thus the reduced order transfer function by adding compensator is given as:

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$$R_2(s) = \frac{0.111(s+0.145)(s+0.13)}{(s^2+0.786s+0.0186)(s+0.164)}$$

By the addition of input time delays for the original system:

$$g_{11}(s) = -0.0114s^8 - 0.5118s^7 - 2.414s^6 - 4.916s^5 - 6.2s^4 - 4.5s^3 - 1.513s^2 - 0.24s + 0.0001$$

$$g_{12}(s) = 0.0193s^8 - 0.112s^7 - 0.111s^6 - 0.418s^5 - 0.324s^4 - 0.318s^3 - 0.161s^2 - 0.0133s + 0.00001$$

$$g_{13}(s) = 0.0021s^8 - 0.22s^7 - 0.885s^6 - 1.315s^5 - 1.211s^4 - 0.836s^3 - 0.314s^2 - 0.014s + 0.000010$$

$$g_{21}(s) = -0.1936s^8 - 0.636s^7 - 2.168s^6 - 4.18s^5 - 5.161s^4 - 3.171s^3 - 1.053s^2 - 0.124s + 0.00009$$

$$g_{22}(s) = -0.0198s^8 + 0.02s^7 + 0.23s^6 - 1.409s^5 - 1.382s^4 - 1.123s^3 - 0.743s^2 - 0.04s + 0.000031$$

$$g_{23}(s) = -0.174s^8 - 0.921s^7 - 2.534s^6 - 4.5s^5 - 4.5s^4 - 2.8s^3 - 1.21s^2 - 0.1503s + 0.00011$$

$$D(s) = s^9 + 10.8s^8 + 43.99s^7 + 94.61s^6 + 125.6s^5 + 108.3s^4 + 56.93s^3 + 15.47s^2 + 1.772s + 0.1988$$

By applying Routh table to the denominator:

s^9	1	43.99	125.6	56.93	1.772
s^8	10.8	94.61	108.3	15.47	0.1988
s^7	35.2298	115.573	55.4976	1.7536	
s^6	59.1804	91.2867	14.9324	0.1988	
s^5	61.2296	46.6084	1.6352		
s^4	46.2383	13.3519	0.1988		
s^3	28.9275	1.372			
s^2	11.16	0.1988			
s^1	0.8566				
s^0	0.1988				

Thus the reduced order denominator is obtained as follows:

$$E_2(s) = 11.16s^2 + 0.8566s + 0.1988$$

By applying Pade Type Model for numerator:

$$C_0 = \frac{1}{e_0} D_0 = \begin{bmatrix} 0.9 & 0.0515 & 0.0545 \\ 0.4795 & 0.1555 & 0.575 \end{bmatrix}$$

$$C_1 = \frac{1}{e_0}(D_1 - e_1 C_0) = \begin{bmatrix} -23313 & -131.2355 & -1385065 \\ -122273 & -39546 & -1474275 \end{bmatrix}$$

$$P_0 = e_0^{'} C_0 = \begin{bmatrix} 0.00018 & 0.0000103 & 0.0000109 \\ 0.0000959 & 0.0000311 & 0.0000115 \end{bmatrix}$$

$$P_1 = C_0 e_1^{'} + e_0^{'} C_1 = \begin{bmatrix} -0.24 & -0.013 & -0.014 \\ -0.125 & -0.0404 & -0.15 \end{bmatrix}$$

Thus the reduced order transfer function is obtained as follows:

$$R_2(s) = \frac{\begin{bmatrix} -0.24s + 0.00018 & -0.013s + 0.0000103 & -0.014s + 0.000010 \\ -0.125s + 0.0000959 & -0.0404s + 0.0000311 & -0.15s + 0.0000115 \end{bmatrix}}{11.16s^2 + 0.8566s + 0.1988}$$

By adding compensator to the reduced order system (Root locus lead compensator):

$$R(s) = \frac{-0.2893s + 0.0002}{11.16s^2 + 0.08566s + 0.1988} * \frac{(s+z_c)}{(s+p_c)}$$

(For 1st output)

$$R(s) = \frac{-0.3154s + 0.00013}{11.16s^2 + 0.08566s + 0.1988} * \frac{(s+z_c)}{(s+p_c)}$$

(For 2nd output)

And the poles of the system are given as follows:
$-0.0423 \pm j0.134$

$$s_d = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$$

$$s^2 + 2\xi\omega_n s + \omega_n^2 = s^2 + 0.0846s + 0.02$$

$$\omega_n = 0.141$$

$$\xi = 0.299$$

The desired value of the reduced order system be

$$\xi = 0.8$$

The values of the compensated system are given as:
$z_c = 0.0283$ *and* $p_c = 0.282$

Thus the reduced order transfer function by adding compensator is given as:

$$R(s) = \frac{-0.2893s + 0.0002}{11.16s^2 + 0.08566s + 0.1988} * \frac{(s+0.0283)}{(s+0.282)}$$

(For 1st output)

$$R(s) = \frac{-0.3154s + 0.00013}{11.16s^2 + 0.08566s + 0.1988} * \frac{(s+0.0283)}{(s+0.282)}$$

(for 2nd output)

Simulation Results: The simulation results for the MIMO compensated system without time delay are given as follows:

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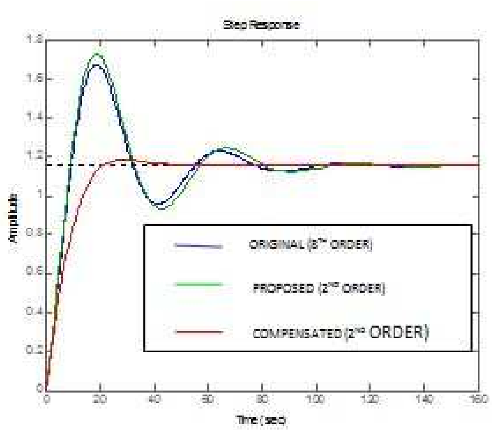

Fig 7. Compensator for without time delay system (for 1st output).

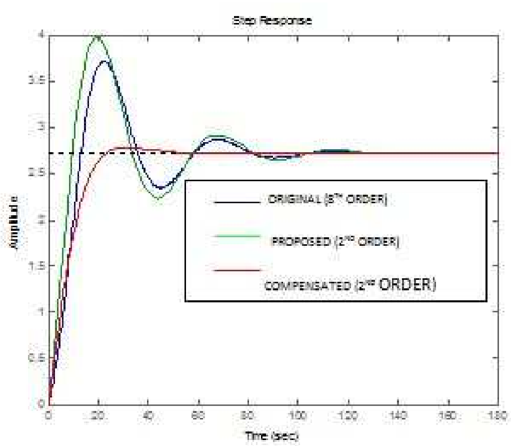

Fig 8: Compensator for the without time delay system (for 2nd output)

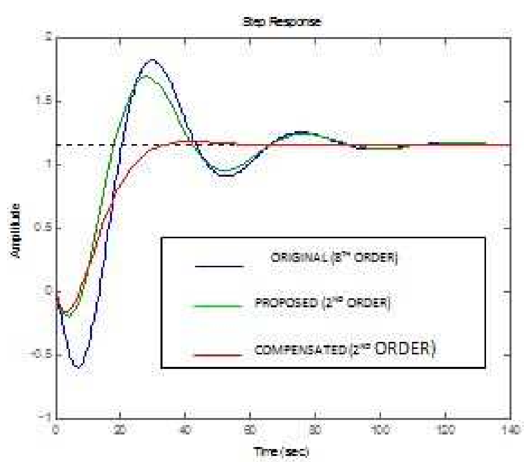

Fig 9: Compensator for with time delay system (for 1st output

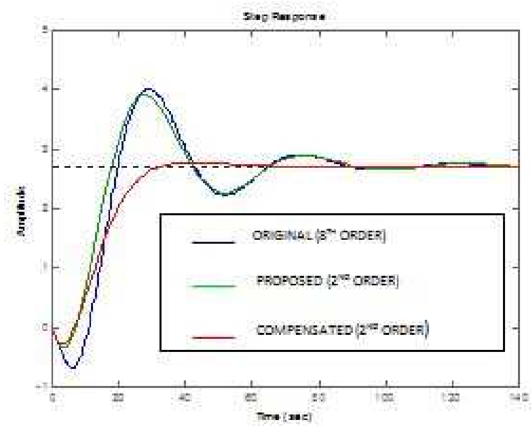

Fig 10: Compensator for with time delay system (for 2noutput).

Comparison of Proposed Method With Other Existing Methods

Example 3

Consider the 4th order original transfer function is given by

$$G(s) = \frac{\begin{bmatrix} g_{11}(s) & g_{12}(s) \\ g_{21}(s) & g_{22}(s) \end{bmatrix}}{D(s)}$$

Where
$g_{11}(s) = 28s^3+496s^2+1800s+2400$
$g_{12}(s) = 30s^3+488s^2+900s+2000$
$g_{21}(s) = 12s^3+528s^2+1440s+4320$
$g_{22}(s) = 24s^3+396s^2+1220s+3200$

and $D(s) = 2s^4+36s^3+204s^2+360s+240$

$G_1(s) = g_{11}(s) + g_{12}(s)$
$G_2(s) = g_{21}(s) + g_{22}(s)$

Where

$$G_1(s) = \frac{58s^3+984s^2+2700s+4400}{2s^4+36s^3+204s^2+360s+240};$$
$$G_2(s) = \frac{36s^3+924s^2+2660s+7520}{2s^4+36s^3+204s^2+360s+240}$$

The reduced second order models obtained by using proposed method is:

$$R_1(s) = \frac{1839s+4400}{184s^2+313s+240}$$ (for 1st output)

$$R_2(s) = \frac{1189s+7520}{184s^2+313s+240}$$ (for 2nd output)

The reduced 2nd order models obtained by using Continued Fraction Expansion method

$$N_1(s) = \frac{16.7s + 4.2}{s^2 + 1.156s + 0.241}$$ (for 1st output)

$$N_2(s) = \frac{28s + 7.4}{s^2 + 1.156s + 0.241}$$ (for 2nd output)

The reduced 2nd order models obtained by using Matrix Cauer form method by R.Prasad

For 1st output: $$P_1(s) = \frac{47.32s + 10}{s^2 + 1.156s + 0.241}$$

For 2nd output: $$P_2(s) = \frac{26.3s + 9.29}{s^2 + 1.156s + 0.241}$$

The step responses of the reduced models obtained by proposed method, continued fraction method, Matrix Cauer form method are shown.

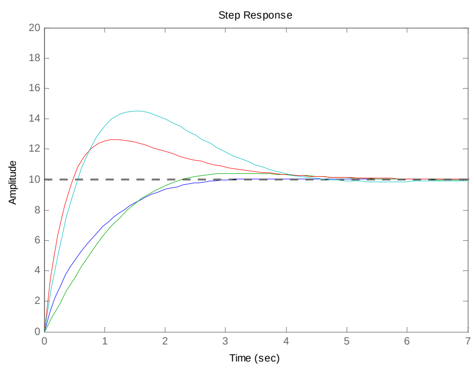

Fig 11. Comparison graphs for existing methods.

Conclusion

In this paper, a new method is proposed for the reduction of high order continuous-time delay systems. The proposed method uses the application of Matrix Pade type model reduction method for obtaining the numerator and Routh table for obtaining the denominator of the reduced order models. This proposed new method overcomes the drawbacks of the some of the existing methods of continuous time systems reduction. The proposed model reduction technique is used for the stability analysis and root locus lead compensator for high-order continuous–time systems is designed.

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